6 Application: Combinatorial Systems

We now turn our attention from the constraining power of the GR operator to its ability to *generate* a language through its second operand. If the first operand of the GR operator is the universal language, the second operand specifies strings that remain. In comparison, if a logical formula ϕ_1 is a tautology, the truth value of the material implication $\phi_1 \rightarrow \phi_2$ depends completely on the right-hand side ϕ_2. Accordingly, any language $X \subseteq \Pi^*$ can be passed through the GR operation unchanged as follows: $X = [\Pi^* \overset{\Pi,0,M}{\Rightarrow} X]$.

6.1 String Coverings with a Lexicon

The things get very interesting when we modify the first operand by adding into it a diamond that occurs before an *arbitrary* character position. This changes a lot: a string in the second operand would now be passed through the GR operation only if its *every* character is disjunctively preceded by a hidden diamond:

$$X=[\nu'_{1,1}(\Pi^*)\overset{\Pi,1,M}{\Rightarrow}\nu'_{\star,1}(X)] \text{ where } \nu'_{1,j}(W)=d^{-1}_{\{\diamond_1,\ldots,\diamond_j\}}(W)\cap(\epsilon\cup(\Pi^*M)\Pi^+) \quad (31)$$
$$\text{and } \nu'_{\star,j}(W)=d^{-1}_{\{\diamond_1,\ldots,\diamond_j\}}(W)\cap(\epsilon\cup(\Pi^*M)^*\Pi^+).$$

The same effect is captured by adding two diamonds that surround each character and empty string on both operand languages:

$$X=[\Pi^*\nu_{2,1}(\epsilon\cup\Pi)\Pi^*\overset{\Pi,2,M}{\Rightarrow}\nu_{\star,1}(X)] \text{ where } \nu_{2,j}(W)=d^{-1}_{\{\diamond_1,\ldots,\diamond_j\}}(W)\cap(\Pi^*\diamond_j\Pi^*\diamond_j\Pi^*)$$
$$\text{and } \nu_{\star,j}(W)=d^{-1}_{\{\diamond_1,\ldots,\diamond_j\}}(W). \quad (32)$$

Free Contexts We can now elaborate the right-hand side of the GR and add there left and right contexts $\#\Pi^*\_\Pi^*\#$:

$$X'=[\nu'_{1,1}(\Pi^+)\overset{\Pi,1,M}{\Rightarrow}\Pi^*\nu'_{\star,1}(X)\Pi^*]=[\Pi^*\nu_{2,1}(\Pi)\Pi^*\overset{\Pi,2,M}{\Rightarrow}\Pi^*\nu_{\star,1}(X)\Pi^*]. \quad (33)$$

Every string in the result X' will now have a *string covering* that consists of possibly overlapping factors X. For example, if $X = \{\texttt{autom}, \texttt{mate}, \texttt{eria}\}$ then X' contains such strings as `automate`, `materia` and `automateriautom`. Accordingly, we have defined a simple combinatorial system. If we want, we can concatenate a unique *sentinel* symbol $\sigma \in \Pi$ to the end of the lexicon $X \subseteq (\Pi \backslash \sigma)^*$ in order to generate $X' = (X\sigma)^*$ *i.e.* a set of strings covered with non-overlapping factors taken from set $X\sigma$.

Problem 2. Can coverings be used to describe allomorph selection, nonconcatenative morphotactics, interdigitation and multi-component rewriting?

Problem 3. Can $\nu'_{\star,j}$ or $\nu_{\star,j}$ be used on the left hand side of an extended GR operator that would still preserve star-freeness? How the change interacts with star-freeness, automata size and applications?